

# Upscaling of Thermo-hydro-mechanical Coupling in Deep Oil Reservoir

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**Abstract:** As a new kind of oil resource, deep oil reservoirs have gained much attention because of large geological reserve. However, since deep reservoirs exhibit strong multiscale characteristics, it is hard to reveal flow mechanisms from only one length scale. So we present an upscaling method to build a full-scale thermo-hydro-mechanical coupling model for deep oil reservoirs in order to study the behaviours of oil flow, heat transfer and rock deformation, which will provide theoretical support to the development of deep oil reservoirs. In this paper, the mesoscopic model is upscaled to macroscopic based on theory of homogenization. And then we use the finite element method to calculate effective coefficients. Results show that Darcy's law for the macroscopic scale can be derived from Stokes equations for the mesoscopic scale. The results of effective coefficients are diagonal, which not only corresponds to isotropic condition, but also indicates that fluid flow and heat transfer mainly happen on main directions.

## 1. Introduction

In recent years, the demand of oil and gas of our country is increasing constantly. But due to long-term development of conventional and shallow reservoirs, it is hard to make breakthroughs in them[1]. We need to find new sources of oil and gas to meet our energy demand. Deep reservoirs have gain more attention. There are abundant deep oil and gas in our country[2]. The oil reserve is  $304 \times 10^8$ t and the gas reserve is  $29.12 \times 10^{12}$  m<sup>3</sup>. Deep oil and gas reservoir will be an important field in the future of petroleum industry.

However, the mechanisms of deep reservoirs haven't been fully understand[3]. Because that deep reservoirs are buried deeply underground, they are under conditions of high temperature, high pressure and high stress. So the rock mechanics, flow mechanisms and fracture propagation are all different from those in conventional shallow reservoirs and thermo-hydro-mechanical coupling must be considered[4]. Besides, since there are pores, matrices, fractures and even vugs of different length scales in deep reservoirs[5], we cannot study from only one scale. For example, if we study from macroscopic scale only, then we will omit the influence of microscopic characteristics. If we study from microscopic scale, it is hard to apply the results in macroscopic numerical simulations because of large amount of calculation[6].

As a result, in this paper, we present a multiscale thermo-hydro-mechanical coupling model of deep oil reservoir using an upscaling method. We upscale the thermo-hydro-mechanical coupling model from Darcy-scale to reservoir-scale based on theory of homogenization. We also give the expressions of effective macroscopic coefficients.

## 2. Model description

### 2.1 Mesoscopic model

The non-dimensional mesoscopic thermo-hydro-mechanical coupling model is as follows[7]:

$$\begin{aligned} & \partial_t [\vartheta: e_x(\mathbf{u}_m) + \omega p_m] + C_f \mathbf{v}_m \nabla_x T_m + \varrho \frac{\partial T_m}{\partial t} \\ & = \nabla \cdot (D_m \nabla_x T_m) - \dot{h}_m (T_m - T_F) \quad \text{in } \Omega_m \end{aligned} \quad (1)$$

$$d_F C_F \frac{\partial T_F}{\partial t} + d_F C_F \mathbf{v}_F \nabla T_F = \nabla \cdot (D_F \nabla T_F) + \dot{h}_F (T_F - T_m) \quad \text{in } \Omega_F \quad (2)$$

$$\nabla \cdot \sigma_m = 0 \quad \text{in } \Omega_m \quad (3)$$

$$\sigma_m = C e_x(\mathbf{u}_m) + \mathcal{B} p_m I - \psi T_m I \quad \text{in } \Omega_m \quad (4)$$

$$\nabla \cdot \mathbf{v}_m - \partial_t [\alpha: e_x(\mathbf{u}_m) + \beta p_m - \gamma T_m] = 0 \quad \text{in } \Omega_m \quad (5)$$

$$\mathbf{v}_m = -\frac{k_m}{\eta} \nabla p_m \quad \text{in } \Omega_m \quad (6)$$

$$\nabla \cdot \sigma_F = 0 \quad \text{in } \Omega_F \quad (7)$$

$$\sigma_F = -p_F I + 2\varepsilon^2 \eta e(\mathbf{v}_F) \quad \text{in } \Omega_F \quad (8)$$

$$\nabla \cdot \mathbf{v}_F = 0 \quad \text{in } \Omega_F \quad (9)$$

$C_f, C_F$  are matrix and fracture compressibility;  $T_f, T_s$  are fluid and solid temperature;  $\mathcal{D}_m, \mathcal{D}_F$  are fluid and solid thermal diffusivity;  $\mathbf{u}_m$  is matrix displacement;  $T_0, \Delta T$  is reference temperature and characteristic temperature;  $\mathbf{v}_m, \mathbf{v}_F$  is fluid velocity in matrix and fracture;  $\eta$  is fluid viscosity;  $\sigma_m, \sigma_F$  is matrix and fracture stress;  $C$  is elastic tensor;  $p_m, p_F$  is pressure;  $I$  is unit vector;  $\Omega_f, \Omega_s, S$  is fluid domain, solid domain and interface of fluid and solid.

## 2.2 Asymptotic expansion and upscaling procedure

Take dimensionless displacement  $\mathbf{u}_m$  for example. We write all the dimensionless variables in asymptotic expansion forms as follows:

$$\mathbf{u}_m(x, t) = \mathbf{u}_m^0(x, y, t) + \varepsilon \mathbf{u}_m^1(x, y, t) + \varepsilon^2 \mathbf{u}_m^2(x, y, t) + \dots \quad (10)$$

$Y_m$  and  $Y_F$  is matrix and fracture domain in a microscopic REV.  $Y$  is the period which is assumed to be 1 in this paper. We replace into dimensionless equations, then we put the terms of same order of  $\varepsilon$  together.

For  $\varepsilon^{-2}$ , we have:

$$\nabla_y p_m^0 = 0 \quad \text{in } Y_m \quad (11)$$

$$\nabla_y p_F^0 = 0 \quad \text{in } Y_F \quad (12)$$

$$\text{div}_y [C e_y(\mathbf{u}_m^0)] = 0 \quad \text{in } Y_m \quad (13)$$

$$C e_y(\mathbf{u}_m^0) \cdot \mathbf{n} = 0 \quad \text{on } S \quad (14)$$

$$D_m \Delta_y T_m^0 = 0 \quad \text{in } Y_m \quad (15)$$

$$D_F \Delta_y T_F^0 = 0 \quad \text{in } Y_F \quad (16)$$

The only possible solutions for above equations are:

$$p_m^0(x, y, t) = p_F^0(x, y, t) = p^0(x, t) \quad (17)$$

$$\mathbf{u}_m^0(x, y, t) = \mathbf{u}_m^0(x, t) \quad (18)$$

$$T_m^0(x, y, t) = T_m^0(x, t) \quad (19)$$

$$T_F^0(x, y, t) = T_F^0(x, t) \quad (20)$$

We can see that macroscopic pressure, displacement and temperature are independent of fast spatial variable  $y$ , which means they do not vary in microscopic view.

For  $\varepsilon^{-1}$  in temperature field, we have:

$$D_m \operatorname{div}_y (\nabla_y T_m^1 + \nabla_x T_m^0) = 0 \quad \text{in } Y_m \quad (21)$$

$$D_F \operatorname{div}_y (\nabla_y T_F^1 + \nabla_x T_F^0) = 0 \quad \text{in } Y_F \quad (22)$$

The solution of above equation can be written as:

$$T_m^1(x, y, t) = \Theta_{mj}(y) \frac{\partial T_m^0}{\partial x_j}(x, t) \quad (23)$$

$$T_F^1(x, y, t) = \Theta_{Fj}(y) \frac{\partial T_F^0}{\partial x_j}(x, t) \quad (24)$$

For  $\varepsilon^{-1}$  in stress field, we have:

$$\nabla_y \cdot C[e_x(\mathbf{u}_m^0) + e_y(\mathbf{u}_m^1)] = 0 \quad (25)$$

For linear problem, we can write the solution in the following linear form:

$$\mathbf{u}_m^1 = \zeta(y) e_x(\mathbf{u}_m^0) - \zeta(y) (1 + \mathcal{B}) p^0 I + \varsigma(y) T_m^0 I \quad (26)$$

Finally, for  $\varepsilon^0$ , we first have:

$$\begin{cases} \nabla_x \cdot \{C[e_x(\mathbf{u}_m^0) + e_y(\mathbf{u}_m^1)] + \mathcal{B} p^0 I - \psi T_m^0 I\} \\ + \nabla_y \cdot \{C[e_x(\mathbf{u}_m^1) + e_y(\mathbf{u}_m^2)] + \mathcal{B} p_m^1 I - \psi T_m^1 I\} = 0 \quad \text{in } Y_m \\ C[e_x(\mathbf{u}_m^1) + e_y(\mathbf{u}_m^2)] + \mathcal{B} p_m^1 I - \psi T_m^1 I = -p_F^1 I + 2\eta e_y(\mathbf{v}_F^0) \quad \text{on } S \end{cases} \quad (27)$$

Integrate both side, we can get macroscopic momentum balance equation:

$$\nabla_x \cdot [C^{eff} e_x(\mathbf{u}_m^0) + \mathcal{B}^{eff} p^0 - M^{eff} T_m^0] = 0 \quad (28)$$

Then we have:

$$\partial_t \{\vartheta: [e_x(\mathbf{u}_m^0) + e_y(\mathbf{u}_m^1)]\} + \omega \frac{\partial p^0}{\partial t} + C_f \mathbf{v}_m^0 (\nabla_x T_m^0 + \nabla_y T_m^1) + \varrho \frac{\partial T_m^0}{\partial t} \quad (29)$$

$$= \nabla_x \cdot [D_m (\nabla_x T_m^0 + \nabla_y T_m^1)] + \nabla_y \cdot [D_m (\nabla_x T_m^1 + \nabla_y T_m^2)] - \hbar_m (T_m^0 - T_F^0)$$

$$d_F C_F \frac{\partial T_F^0}{\partial t} + d_F C_F \mathbf{v}_F^0 (\nabla_x T_F^0 + \nabla_y T_F^1) - \hbar_F (T_m^0 - T_F^0) \quad (30)$$

$$= \nabla_x \cdot [D_F (\nabla_x T_F^0 + \nabla_y T_F^1)] + \nabla_y \cdot [D_F (\nabla_x T_F^1 + \nabla_y T_F^2)]$$

We also integrate on both side, then we can get macroscopic energy balance equation:

$$\langle \vartheta [I + e_y(\zeta)] \rangle \frac{\partial e_x(\mathbf{u}_m^0)}{\partial t} + \langle [\omega - \vartheta e_y(\zeta) (1 + \mathcal{B})] \rangle \frac{\partial p^0}{\partial t} + C_f \langle \mathbf{v}_m^0 \rangle \nabla_x T_m^0 \quad (31)$$

$$+ \langle [\varrho + \vartheta e_y(\varsigma)] \rangle \frac{\partial T_m^0}{\partial t} = \nabla_x \cdot (D_m^{eff} \nabla_x T_m^0) - (1 - \Phi_F) \hbar_m (T_m^0 - T_F^0) \quad \text{in } \Omega_m$$

$$d_F C_F \frac{\partial T_F^0}{\partial t} + d_F C_F \mathbf{v}_F^0 \nabla_x T_F^0 = \nabla_x \cdot (D_F^{eff} \nabla_x T_F^0) + \hbar_F (T_m^0 - T_F^0) \quad \text{in } \Omega_F \quad (32)$$

Finally, for mass balance equations, we have:

$$\begin{cases} \frac{\eta}{\mathbf{k}_m} \mathbf{v}_m^0 = -(\nabla_x p_m^0 + \nabla_y p_m^1), \nabla_y \cdot \mathbf{v}_m^0 = 0 \text{ in } \Omega_m \\ \nabla_x p_F^0 + \nabla_y p_F^1 - 2\eta \text{div}_x e_y(\mathbf{v}_F^0) = 0, \nabla_y \cdot \mathbf{v}_F^0 = 0 \text{ in } \Omega_F \\ \mathbf{v}_m^0 + \frac{\partial \mathbf{u}_m^0}{\partial t} = \mathbf{v}_F^0 \text{ on } S \\ p_m^1 = p_F^1 \text{ on } S \end{cases} \quad (33)$$

We define  $\mathbf{v}_F^* = \mathbf{v}_F^0 - \frac{\partial \mathbf{u}_m^0}{\partial t} = \mathbf{v}_m^0$

Then the solution of can be written as:

$$\begin{cases} \mathbf{v}_m^0 = -\frac{\varpi_m(y)}{\eta} \nabla_x p^0, p_m^1 = -\pi_m(y) \nabla_x p^0 \\ \mathbf{v}_F^* = -\frac{\varpi_F(y)}{\eta} \nabla_x p^0, p_F^1 = -\pi_F(y) \nabla_x p^0 \end{cases} \quad (34)$$

For total velocity, we have:

$$\langle \mathbf{v}_T^0 \rangle = \langle \mathbf{v}_m^0 \rangle_{\Omega_m} + \langle \mathbf{v}_F^* \rangle_{\Omega_F} = -\frac{1}{\eta} \left[ \int_{Y_m} \varpi_m(y) dy + \int_{Y_F} \varpi_F(y) dy \right] \nabla_x p^0 \quad (35)$$

Define macroscopic permeability tensor, we have:

$$k_{ij}^{eff} = \left\{ \int_{Y_m} [\varpi_m^i(y)]_j dy + \int_{Y_F} [\varpi_F^i(y)]_j dy \right\} \quad (36)$$

Since  $\mathbf{v}_m^0 = \mathbf{v}_f^0 - \Phi \frac{\partial \mathbf{u}_m^0}{\partial t}$ , we can get:

$$\langle \mathbf{v}_T^0 \rangle = \langle \mathbf{v}_f^0 \rangle_{\Omega_m} + \langle \mathbf{v}_F^0 \rangle_{\Omega_F} - [\Phi(1 - \Phi_F) + \Phi_F] \frac{\partial \mathbf{u}_m^0}{\partial t} = \langle \mathbf{v}_T^0 \rangle^p - \Phi^{eff} \frac{\partial \mathbf{u}_m^0}{\partial t} \quad (37)$$

$\langle \mathbf{v}_T^0 \rangle^p$  is fluid velocity in total pores.

For  $\varepsilon^0$ , we have:

$$\nabla_x \cdot \mathbf{v}_F^0 + \nabla_y \cdot \mathbf{v}_F^1 = 0 \quad (38)$$

Integrate on both side, then wen can get macroscopic mass balance equation as follows:

$$\nabla_x \cdot \langle \mathbf{v}_T^0 \rangle + \alpha^{eff} \frac{\partial e_x(\mathbf{u}_m^0)}{\partial t} - \beta^{eff} \frac{\partial p^0}{\partial t} + \gamma^{eff} \frac{\partial T_m^0}{\partial t} = 0 \quad (39)$$

Now we have the whole macroscopic thermo-hydro-mechanical coupling model derived from mesoscopic view.

### 3. Effective coefficient calculation and discussions

(1) Elastic tensor

$$C^{eff} = \begin{bmatrix} 1.173 & 0.5425 & 0.5425 & -6.301e^{-18} & -3.772e^{-18} & 6.924e^{-18} \\ 0.5425 & 1.173 & 0.5425 & -2.608e^{-17} & -8.391e^{-18} & -3.711e^{-17} \\ 0.5425 & 0.5425 & 1.173 & 2.554e^{-17} & -1.963e^{-17} & -2.910e^{-18} \\ -6.301e^{-18} & -2.608e^{-17} & 2.554e^{-17} & 0.5403 & -2.466e^{-18} & 9.062e^{-19} \\ -3.772e^{-18} & -8.391e^{-18} & -1.963e^{-17} & -2.466e^{-18} & 0.5403 & 7.350e^{-17} \\ 6.924e^{-18} & -3.711e^{-17} & -2.910e^{-18} & 9.062e^{-19} & 7.350e^{-17} & 0.5403 \end{bmatrix}$$

## (2) Permeability tensor

$$\mathcal{K}^{eff} = \begin{bmatrix} 1.735e^{-3} & 8.479e^{-23} & 2.551e^{-23} \\ 3.906e^{-22} & 1.735e^{-3} & 3.844e^{-23} \\ 6.560e^{-23} & 1.128e^{-22} & 1.735e^{-3} \end{bmatrix}$$

We can see that coefficients matrices are diagonal, which correspond to isotropic case. The permeability of main directions is much larger than those in minor directions. That's because pressure is posed on main directions.

## 4. Conclusion

In this paper, we upscale the thermo-hydro-mechanical coupling models in deep oil reservoir from Darcy-scale to macro-scale using theory of homogenization. Microscopic parameters are incorporated in macroscopic models and effective coefficients. We can consider that upscaling method built up a bridge between models of different scales to reflect microscopic information to macroscopic models.

Through upscaling, we derive Darcy's law from microscopic Stokes equation. We also present macroscopic energy balance, mass balance and momentum balance equations. Besides, we calculate some typical effective coefficients in the third section. Isotropic case is mainly discussed. We can see that the results match well with the geometry, which also show the correctness of our models.

Our future job will contain numerical simulations of deep oil reservoir to analyze how microscopic parameters like pore sizes, shape, distributions as well as fractures' characters will affect macroscopic flow behavior and rock mechanics.

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